Spectroscopy of charmed D meson and form factor of B $\rightarrow$ D* lnu in LQCD

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Semileptonic B decays are of primary importance since, for example, they participate very strongly in the accurate determination of the CKM matrix element V_{cb} which represents a test of the Standard Model. However, there are many puzzling features associated with the semileptonic $b \to c$ data, which have appeared during the last ten years such as the so-called "1/2 versus 3/2 puzzle" which corresponds to the difference between theoretical predictions and experimental measurements of semileptonic branching ratios of $\bar{B} \to D^{**}\ell\nu$.
In many theoretical approaches (HQET, heavy quark expansion, quark model, Lattice QCD with quenched calculation, etc...), branching ratios corresponding to the decay $\bar{B} \to D^{**}\ell\nu$ were calculated using the infinite mass limit. That is the reason why, in order to address the aforementioned questions, we propose to determine for the first time the physical parameters and then the branching ratios using "real" charmed quarks having a finite mass.

1.0.1 Spectroscopy

In a heavy meson rest frame, the total angular momentum reads :

$$\boxed{\vec{J} \; = \; \vec{s}_h \; + \; \vec{j}}$$

$$\text{where} \begin{cases} \vec{s}_h & \text{spin of the heavy quark} \\ \vec{j} & \text{angular momentum of the light cloud} \end{cases}$$

In quark model, $\vec{j}$ has the following expression

$$\boxed{\vec{j} \; = \; \vec{s}_l \; + \; \vec{l}}$$

$$\text{where} \begin{cases} \vec{s}_l & \text{spin of the light quark} \\ \vec{j} & \text{orbital angular momentum of the light cloud} \end{cases}$$

In the infinite mass limit there are no interactions involving the heavy quark spin. Therefore s_h is a conserved quantity and since $\vec{J}$ is also conserved, it is convenient to use j^P as an index to classify static-light meson states where P is the parity.

P wave states ($l = 1$) : This case gathers the first orbital excitations of heavy mesons M and they are represented as M^{**}. j has two values 1/2 and 3/2. Each of these values form a doublet. In the case where the heavy quark is the charm c, we obtain the D^{**} represented in the following table :

doublet	J^P values	notation
$j^P = 1/2^+$	0^+	D_0^*
	1^+	D_1^*
$j^P = 3/2^+$	1^+	D_1
	2^+	D_2^*

S wave states ($l = 0$) : Heavy mesons satisfying this property are classified as :

doublet	J^P values	notation
$j^P = 1/2^-$	0^-	D
	1^-	D^*

finite quark masses : a significant difference with respect to the static approximation is that there is no heavy spin degeneracy anymore, i.e. there are two S wave states and there are four P wave states (notation D_J^j : $D_0^{1/2}$, $D_1^{1/2}$, $D_1^{3/2}$, $D_2^{3/2}$). The angular momentum of the light cloud j is not a good quantum number anymore, i.e. states must be labeled by their total angular momentum J^P.

1.1 Lattice QCD (LQCD)

At short distances (or equivalently at high energies) the quarks interact weakly, so that it is possible to study the theory of the strong interaction (Quantum Chromodynamics QCD) with perturbative techniques. But the growth of the coupling constant in the infrared - the flip side of asymptotic freedom- requires the use of non-perturbative methods to determine the low energy properties of QCD. This is the case for many observables playing important role in the context of Flavor Physics, as for example the form factors, the decay constants, and numerous matrix elements involved in meson mixings.
The only way to determine physical observables in a non perturbative way and starting from first principles is Lattice QCD or LQCD [1]. Lattice gauge theory, proposed by Wilson in 1974, is a way to regularize Field Theory in which the continuum and infinite space-time is replaced with a discretized grid of points $x \in \{n\}$ in a finite volume, of extent L in space and T in time and a is the lattice spacing. Quark fields are placed on sites and gauge fields on the links between sites. These links represent the gluon field on the lattice (or "*gauge link*")

1. for further reading about Lattice QCD, I refer the reader to [1, 2]

$U_\mu(x)$ which is an element of the group $SU(3)$.
The simplest expression which in the continuum limit reduces to the gluon action has been found by Wilson and expressed in terms of the *plaquettes* $U_{n;\mu\nu}$, defined as the ordered product of the four links variables lying over the border of the square defined by the points x and $x + \hat{\mu} + \hat{\nu}$:

$$U_{x;\mu\nu} \equiv U_{x;\nu} U_{x+\hat{\mu};\nu} U^{\dagger}_{x+\hat{\nu};\mu} U^{\dagger}_{x;\nu}$$

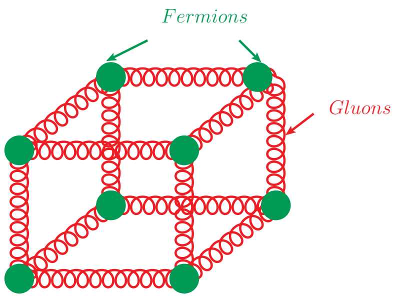
The Wilson action for gauge fields reads

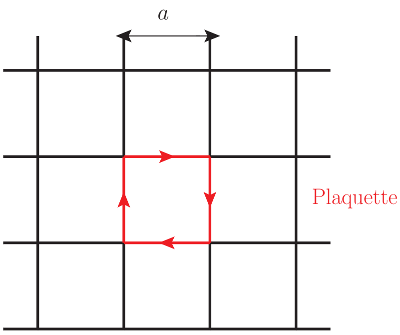
$$S_G \;=\; \frac{\beta}{3} \sum_{x\in\{n\}} \sum_{\mu<\nu} \mathrm{Re\ tr}\,[1 - U_{\mu\nu}(x)] \qquad \beta = 6/g^2$$

which is a gauge invariant quantity. In general, Wilson discretized action of QCD is given by

$$S_{LQCD} \equiv \sum_f S_F + S_G$$

where S_F and S_G are respectively the discretized ***fermion*** and the ***gluon actions***.





1.1.1 Simulation setup

The ***gauge action*** used in our simulation is tree-level Symanzik improved [3] with $\beta = 3.9$ corresponding to a lattice spacing $a = 0.0855$fm.

$$\begin{aligned} S_G \;&=\; \frac{\beta}{6}\,(b_0 \sum_{x,\mu,\nu} Tr(1 - P^{1\times 1}(x;\mu,\nu)) \\ &+\; b_1 \sum_{x,\mu\nu} Tr(1 - P^{1\times 2}(x;\mu,\nu)) \end{aligned}$$

where $b_0 = 1 - 8b_1$ and $b_1 = -1/12$.
The ***fermionic action*** used in our simulation is Wilson Twisted-mass Lattice QCD (tmLQCD) with two flavors of mass-degenerate quarks, tuned at *maximal twist* in the way described in full details in Ref. [4] :

$$S_F[\chi,\bar{\chi},U] \;=\; a^4 \sum_x \bar{\chi}(x)(D_W \;+\; i\mu_q\gamma_5\tau_3)\chi(x)$$

where

$$D_W \;=\; \frac{1}{2}\,(\gamma_\mu(\nabla_\mu \;+\; \nabla^*_\mu) - a\nabla^*_\mu\nabla_\mu) + m_0$$

is the *Wilson Dirac operator*, $\chi = (\chi^{(u)},\ \chi^{(d)})$ are the fermionic fields in the twisted basis. They are obtained from an axial rotation of $\psi, \bar{\psi}$ (fermion fields in the physical basis where the continuum QCD action takes the standard form) [5]. *TmLQCD action* reduces the discretization error and improves the current computation. ∇_μ and ∇^*_μ are the standard gauge covariant forward and backward derivatives, m_0 and μ_q are the bare untwisted and twisted quark masses.
We use an ensemble of 100, $N_f = 2$ flavor, $24^3 \times 48$ Wilson Twisted mass gauge configurations produced by the European Twisted Mass Collaboration (ETMC).
We consider 5 valence quark masses : one light quark mass ($a\mu_l = 0.0085$), one corresponding to the charm quark ($a\mu_c = 0.25$) and the others to the beauty quark mass ($a\mu_b = 0.34, 0.45, 0.67$). Here, we perform *partially quenched* calculations since the masses of the valence quarks (u, d, c, b) and the sea quarks (u, d) differ.

1.2 Charmed meson masses in LQCD

The simplest quantities involving fermions that one can compute on the lattice are the masses of hadrons. I first discuss how to construct operators with the correct quantum numbers and their correlation functions. I continue with the analysis of the resulting mesonic correlation functions and discuss how to obtain the corresponding masses.

1.2.1 Meson creation operators

A meson creation operator has the following form :

$$\mathcal{O}(t) \;=\; \bar{\psi}(t,\vec{x}_Q)\,\mathcal{P}_t(\vec{x}_Q,\vec{x}_q)\Gamma\;\psi_q(t,\vec{x}_q)$$

where $\bar{\psi}(t,\vec{x}_Q)$ creates an antiquark at position $\vec{x}_Q$, $\psi_q(t,\vec{x}_q)$ creates a quark at position $\vec{x}_q$ and $\mathcal{P}_t(\vec{x}_Q,\vec{x}_q)\Gamma$ is a suitable combination of gauge links and γ matrices which defines the spin structure of the operator.
The simplest creation operators are called "*local operators*" :

$$\mathcal{O}(t) \;=\; \bar{\psi}(t,\vec{x}_Q)\;\Gamma\;\psi_q(t,\vec{x}_q)$$

where the quantum numbers are determined by the choice of the gamma matrix Γ. Local operators allow access only to the set $J^{PC} = (0,1)^{\pm\pm}$. In order to consider higher spins, one must consider "*nonlocal operators*" [6] and, to this end, one strategy is based on group theory and particularly on the irreducible repre-

J	Subduced representations
0	$A_1^{\pm}$
1	$T_1^{\pm}$
2	$E^{\pm} \oplus T_2^{\pm}$
3	$A_2^{\pm} \oplus T_1^{\pm} \oplus T_2^{\pm}$

TABLE 1.1 – Subduced representations tabulated up to $J = 3$ of O_h

sentations of the cubic group O_h (the symmetry group on lattice) : 1-dimensional representation called A, 2-dimensional E and 3-dimensional T. The strategy aims to relate the total angular momentum of the state to the irreducible representations of O_h by looking for what we call the "*Subduced representations*" listed in table 1.1. Now, By choosing different path combination and appropriate choices of Γ one can obtain, for example, different $J = 2$ states which transform under $E^{\pm}$ and $T_2^{\pm}$ representations. Thus, the appropriate operators for $J^P = 2^+$ charmed mesons are :

$$\boxed{E^+} : \begin{cases} \gamma_1 D1 + \gamma_2 D_2 - 2\gamma_3 D_3 \\ \gamma_1 D_1 - \gamma_2 D_2 \end{cases}$$

$$\boxed{T^+} : \begin{cases} \gamma_1 D2 + \gamma_2 D_1 \\ \gamma_1 D_3 + \gamma_3 D_1 \\ \gamma_2 D_3 + \gamma_3 D_2 \end{cases}$$

where D_i are the covariant derivative on lattice in direction i.

1.2.2 Two-point correlation functions

In Lattice QCD, the mesonic masses are determined from the exponential fall of two-point correlation function (or vacuum expectation value) of appropriate meson creation operators $\mathcal{O}(t)$ at large time separation.

$$\begin{aligned} \mathcal{C}^{(2)}(T) &= \langle \Omega | \mathcal{O}^{\dagger}_{t+T} \mathcal{O}_t | \Omega \rangle \\ T \gg 0 &\approx \underbrace{|\langle 0|\mathcal{O}|\Omega\rangle|^2}_{\mathcal{Z}_D} \exp(-\underbrace{(E_0 - E_{\Omega})}_{M_{meson}} T) \end{aligned} \tag{1.1}$$

Masses of $D(J^P = 0^-)$ and $D_0^*(J^P = 0^+)$: when we are looking for charm scalar masses (D_0^*), we do not take into account the scalar correlators only because of the mixing between scalar and pseudoscalar states due a drawback of the tmLQCD action which is the parity violation. So instead, we consider a 2×2 correlation matrix $\mathcal{C}^{(2)}_{jk}(T)$ composed of scalar and pseudoscalar operators in the twisted basis, i.e $\mathcal{O}_j \in \{\bar{\chi}^{(c)}\gamma_5\chi^{(u)},\ \bar{\chi}^{(c)}\chi^{(u)}\}$. Then, we resolve the **Generalized Eigenvalue Problem** (GEVP) [10] in order to find the corresponding eigenvectors and eigenvalues of the system :

$$\sum_k C_{jk}(t) v_k^{(n)}(t, t_0) = \sum_k \lambda^{(n)}(t, t_0) C_{jk}(t_0) v_k^{(n)}(t, t_0)$$

Finally, we obtain the corresponding two masses as a function of time from the ratio of the eigenvalues at consecutive times :

$$\frac{\lambda^{(n)}(t, t_0)}{\lambda^{(n)}(t+1, t_0)} = \frac{e^{-m^{(n)}(t,t_0)\,t} + e^{-m^{(n)}(t,t_0)\,(T-t)}}{e^{-m^{(n)}(t,t_0)\,(t+1)} + e^{-m^{(n)}(t,t_0)\,(T-(t+1))}}$$

The effective mass plateaus shown in figure 1.1 correspond to the soughtmeson masses. The difference bet-

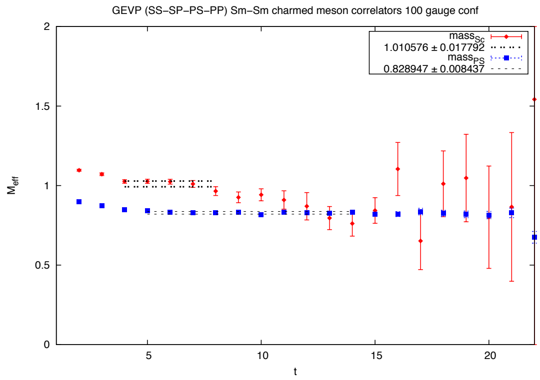


FIGURE 1.1 – Effective masses, of scalar and pseudoscalar charm meson, as a function of time in lattice unit calculated at one value of lattice spacing a.

ween scalar and pseudoscalr masses ($M_{D_0^*} - M_D$), at this value of lattice spacing, differs about 30% to what was found experimentally. But when extrapolated to the continuum, the difference between masses is about 10% and this slight difference between LQCD and experiment comes from to finite lattice spacing effect in LQCD.

1.3 Hadronic matrix elements in LQCD

Hadron spectrum is explored by matrix elements of suitable operators between hadronic states or the vacuum. The matrix elements of vector or axial vector currents between single hadron states lead to the *weak form factor*. Further matrix elements provide information on semileptonic decays, quark and gluon structure functions, etc...

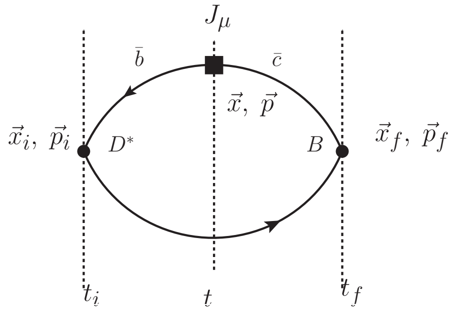


FIGURE 1.2 – Sketch of the valence quark flow in the form factor of the $B \to D^* \ell\nu$.

To access matrix elements on the lattice, one computes the following three- and two-point correlation

(1.1) functions

$$\mathcal{C}^{(3)}(t,t_i,t_f,\vec{p}_i,\vec{p}_f) = \sum_{\text{positions}} \langle \mathcal{O}^{\dagger}_{D^*}(t_f,\vec{x}_f)\ J_\mu(t,\vec{x})\ \mathcal{O}_B(t_i,\vec{x}_i)\rangle \cdot e^{i(\vec{x}-\vec{x}_f)\cdot\vec{p}_f} \cdot e^{-i(\vec{x}-\vec{x}_i)\cdot\vec{p}_i}$$

where $\mathcal{O}^{\dagger}_{D^*}$, $\mathcal{O}_B$ are respectively the creation and annihilation operators of D^* and B mesons, J_μ is the vector or axial current.
From the asymptotic behavior of the three-point correlation function, it is obvious that the removal of the exponential factors can be achieved by considering the ratio

$$\mathcal{R}(t) = \frac{\mathcal{C}^{(3)}(t,t_i,t_f,\vec{p}_i,\vec{p}_f)}{\mathcal{C}^{(2)}_{(B)}(t-t_i,\vec{p}_f)\cdot\mathcal{C}^{(2)}_{(D)}(t_f-t,\vec{p}_i)} \cdot \sqrt{\mathcal{Z}_B}\cdot\sqrt{\mathcal{Z}_D} \qquad (1.2)$$

where $\mathcal{Z}_M = |\langle 0|\mathcal{O}_M|M\rangle|^2$ is obtained from the fit with asymptotic behavior of the two-point correlation functions.

$$\mathcal{C}^{(2)}(t) \xrightarrow[t\to\infty]{} \frac{\mathcal{Z}_M}{2E_M}\,{}^{-E_M t}$$

When the operators in the ratio (1.2) are sufficiently separated, one observes the stable signal (plateau), which is the desired matrix element :

$$\mathcal{R}(t) \xrightarrow[t-t_i\to\infty]{t_f-t\to\infty} \langle D(\vec{p}_f)|(A_\mu,\,V_\mu)|B(\vec{p}_i)\rangle$$

Form factor of $B\to D^*\ell\nu$ at zero recoil ($\vec{p}_i=\vec{0}$) : In order to calculate the form factor at zero recoil of $B\to D^*\,\ell\nu$: $\mathcal{F}_0(1)$, we choose to work in the rest frame of D^* $(\vec{p}_f=\vec{0})$. This form factor is an essential ingredient for the determination of the CKM matrix element Vcb in an exclusive way and is obtained from the corresponding hadronic matrix element $\langle D^*(\vec{0})|(A_\mu,\,V_\mu)|B(\vec{0})\rangle$.
So, using the ratio of the corresponding three point functions over the two-point functions of B and D^* mesons (1.2), we extract the plateaus of hadronic matrix element from the fit in $t\in[4,\ 9]$.

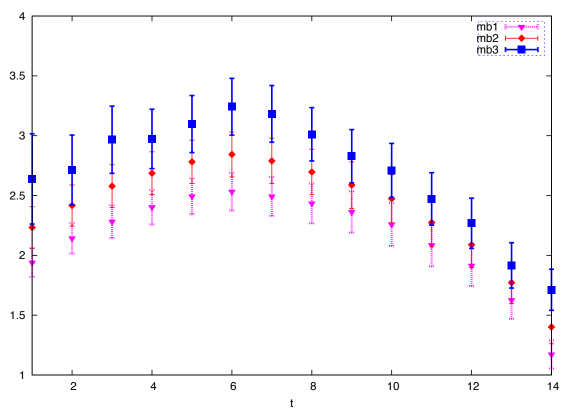


FIGURE 1.3 – Plateaus of the hadronic matrix element for different b quark masses as a function of time in lattice unit calculated at one value of lattice spacing a. $mb3$ corresponds to the heaviest quark mass.

$a\mu_b$	$\mathcal{F}_0(1)$
0.35	0.94 $\pm$0.06
0.45	0.95 $\pm$0.06
0.67	0.96 $\pm$0.07

TABLE 1.2 – $\mathcal{F}_0(1)$ for different values of b quark mass $a\mu_b$

Using [7, 8], the expression of $\mathcal{F}_0(1)$ reads :

$$\mathcal{F}_0(1) = \frac{\langle D^*(\vec{0})|A_i|B(\vec{0})\rangle}{2\sqrt{M_B\cdot M_{D^*}}}$$

Once calculated, this result should be multiplied by the renormalisation constant $Z_A = 0.730$ [9] in order to get the physical value of the form factor. At the moment, the result of table 1.2 has not yet been extrapolated but it does not show any contradiction with what was already found using Lattice QCD with a different fermionic action and different approximations.

Conclusion and perspective : In the last ten years, techniques and algorithms in LQCD had been developed and computing power had been increased and this is really encouraging for Lattice community where some important parameters in B physics are and will be calculated. In the near future, we aim to calculate the form factors of $B\to D_0^*, D_2^*$. We have some preliminary results, not refined yet. When finished, we hope to discuss their impact on the “puzzle 1/2 versus 3/2" and see if it is still confusing or not anymore.

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